ALGEBRAIC AND GEOMETRIC COMBINATORICS ON CONVEX POLYTOPES

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Gabriele Balletti

On the maximal dual volume of a lattice polytope with one interior point

It is well known that the volume of a canonical Fano lattice polytope P (i.e a lattice polytope with one interior lattice point) of some fixed dimension is bounded. A sharp bound has been conjectured, but never been proved. In a joint work with A. Kasprzyk and B. Nill we prove that such sharp bound is valid (and sharp) when the volume of the dual P is considered. As a corollary we have a sharp bound for the volume of reflexive polytopes.

Katy Beeler

Generalized $(0, 1, 2)$*-polytopes*

0/1-polytopes and half-integral polytopes are important polytopal families in linear programming and combinatorial optimization. While 0/1-polytopes have been fairly well studied and, in particular, are known to satisfy the Hirsch conjecture, not much is know about half-integral polytopes. Linearly isomorphic to the half-integral polytopes are the $(0, 1, 2)$-polytopes, polytopes with vertex-coordinates in $\{0, 1, 2\}$. In this talk I will present different generalizations of $(0, 1, 2)$-polytopes and in particular, an enumeration of all $(0, 1, a)$-polytopes in dimension three (polytopes with vertex-coordinates in $\{0, 1, a\}$). Additionally, I will prove that the optimal bound for the diameter of $(0, 1, a)$-polytopes is $\lfloor \frac{3}{2}d \rfloor$, for any dimension d.

Mónica Blanco

Sublattice index of lattice 3-polytopes

Joint work with Francisco Santos

We call a lattice d-polytope P *primitive* if $P \cap \mathbb{Z}^d$ is an affine lattice basis for $\mathbb{Z}^d$. More generally, we call *sublattice index* of P the index, as a sublattice of $\mathbb{Z}^d$, of the affine lattice generated by $P \cap \mathbb{Z}^d$. I will show that the complete classification of non-primitive lattice 3-polytopes is as follows:

- Non-primitive lattice 3-polytopes of width one can have any index, but are easy to classify since all their lattice points lie in two segments.
- Non-primitive lattice 3-polytopes of width larger than one can only have sublattice index 2, 3 or 5 and there are quadratically many for each number of lattice points in P. More precisely: there is a unique polytope of index 5 (a "terminal tetrahedron" of volume 20), and those of indices 2 and 3 fall, apart of a few exceptions with at most eight lattice points, under a few infinite families projecting to a specific (very short) list of lattice polygons.

Finally, we also show that all primitive 3-polytopes, except for two specific "terminal tetrahedra", contain a unimodular tetrahedron.

Federico Castillo

Ehrhart positivity

We call a polytope P positive, if its Ehrhart polynomial has all positive coefficients. In the literature there are different examples using different techniques to prove positivity. We will survey some of these results. It turns out, through work of Danilov, McMullen, and others, that there is an interpretation of the coefficients relating to the normalized volume of faces. We try to make this relation more explicit in the particular case of the regular permutohedron. The goal, long term, is to prove positivity for all generalized permutohedra. This is joint work with Fu Liu.

Simon Hampe

The intersection ring of matroids

We study a particular graded ring structure on the set of all loopfree matroids on a fixed labeled ground set, which occurs naturally in tropical geometry. The product is given by matroid intersection and the additive structure is defined by assigning to each matroid the indicator vector of its chains of flats. This ring has many striking properties, which we will outline in this talk. First and foremost, Derksen's G-invariant, and thus many more matroid invariants (e.g., the Tutte polynomial), induce linear maps from this ring. One can show that the matroids form the vertices of a polytope in the the ring, so many extremal problems in matroid theory can be translated to optimization problems. The talk will also include a very brief excursion into tropical geometry, demonstrating the geometric intuition behind these ideas.

Jan Hofmann

The finiteness threshold width of lattice polytopes

Trying to classify lattice polytopes, it seems natural to try to enumerate all of them by fixing the dimension and the number of lattice points. In this talk we will see this work marvellously in dimensions 1 and 2, but fail miserably from dimension 3 on. One possible remedy for this is the finiteness threshold width of lattice polytopes, which is defined as the width $w^{\infty}(d)$ beyond which there are only finitely many d-dimensional lattice polytopes with a given number of lattice points and width $> w^{\infty}(d)$. In this talk we will see that $w^{\infty}(d)$ is finite and $w^{\infty}(4) = 2$. This is joint work with Mónica Blanco, Christian Haase and Francisco Santos.

Florian Kohl

Transfer-Matrix Methods meet Ehrhart Theory

Counting (proper) colorings of graphs is a classical problem in combinatorics with connections to many different fields. In this talk, we want to examine proper k-colorings of graphs of the form $G \times P_n$, where G is any graph, and where P_n is the path graph on n nodes. There are two special cases, namely (1) where k is fixed but not n, and (2) where n is fixed but not k. In (1), the number of colorings can be determined using the transfer-matrix method, and in (2), the number of colorings can be counted using the chromatic polynomial/Ehrhart theory. We will use the symmetry of $G \times P_n$ to combine the two methods to get explicit formulas (depending on k and n) counting the number of colorings and we will give a restricted version of the famous reciprocity theorem for the chromatic polynomial. Furthermore, we will describe the doubly asymptotic behavior of graphs $G \times C_n$, where C_n is the cycle graph with n nodes as both n and k go to infinity. If time permits, we will explain how a similar method can be used to determine Ehrhart polynomials of order polytopes of a special class of posets. This is joint work with Alexander Engstroem.

Lauri Loiskekoski

Separation in the graph of a simple polytope

Separation in graphs is related to graphs being expanders. A conjecture by Kalai generalizes the planar separation theorem to simple polytopes. It states that the graph of a simple d-polytope can be separated to two roughly equal parts by removing $O(n^{\frac{d-2}{d-1}})$ vertices. We provide a counterexample to this conjecture. This is joint work with Günter Ziegler.

Marie Meyer

Polyhedral Constructions using the Laplacian Matrix

In this talk we will focus on two polyhedral constructions arising from the well studied Laplacian matrix of a graph. We will first look at a polyhedral proof of the Matrix Tree Theorem presented by Dall and Pfeifle, which defines zonotopes based on the columns of the Laplacian matrix of a graph. Then I will present a new polyhedral construction, namely taking the convex hull of the columns of the Laplacian matrix to form a simplex. Finally I will discuss known properties of these simplices according to graph type.

McCabe Olsen

Hilbert bases for Gorenstein lecture hall cones

Lecture hall partitions and related combinatorial objects have been the topic of much recent study. The focus of this talk will be ongoing work in classifying the Hilbert bases of certain Gorenstein lecture hall cones. In particular, we will discuss the $1 \bmod k$ cones and ℓ-sequence cones.

Benjamin Schröter

Split and k-Splits of the Hypersimplex

The study of regular subdivisions is an active field in mathematics. There are many constructions for finest subdivisions, but only few for coarsest subdivisions. Coarse subdivisions of the hypersimplex are of special interest in tropical geometry, since tropical linear spaces are dual to such subdivisions. One class of coarsest subdivisions are k-splits, invented by Sven Herrmann. I shall present a classification of all $k$splits for the hypersimplex and their strong relation to matroid theory. The study of 2-splits leads to a new large class of matroids. As an application we give a dimension bound for the moduli space of tropical linear spaces and a construction for coarsest tropical linear spaces which are non realizable.

Hannah Sjöberg

Projections of the set of flag vectors of 4-polytopes

The set of f-vectors of 3-dimensional polytopes was fully described by Steinitz in 1922. To characterize the set of f-vectors for higher dimensions is an open problem. For dimensions higher than 3, the notion of *flag vectors* is useful. The flag vector counts chains of faces. In this talk we will look at the cone of flag vectors of 4-polytopes, projections of the set of flag vectors of 4-polytopes and generalizations to higher dimensions.

Liam Solus

Learning Bayesian Networks via Edge Walks on DAG Associahedra

The focus of this talk will be an application of convex polytopes to causal inference. Graphical models based on directed acyclic graphs (DAGs), also known as Bayesian networks, are used to model complex cause-and-effect systems across a vast number of research areas including computational biology, epidemiology, sociology, and environmental management. A DAG model is family of joint probability distributions over the nodes of a DAG G that entail a set of conditional independence (CI) relations encoded by the nonedges of G. A fundamental problem in causality is to learn an unknown DAG G based only on a set of observed conditional independence relations. Since multiple DAGs can encode the same set of CI relations, a property termed Markov equivalence, the goal is to identify efficient algorithms that consistently recover a DAG within the correct Markov equivalence class. In this talk, we will describe a pair of greedy algorithms for DAG model selection that operate via edge walks on a family of generalized permutohedra called DAG associahedra. We will present consistency guarantees for these algorithms, and compare them with the more classical approaches to Bayesian model selection in both efficiency and strength of satisfied identifiability assumptions.

Yusuke Suyama

Toric Fano varieties associated to building sets

We give a necessary and sufficient condition for the nonsingular projective toric variety associated to the graphical building set of a finite simple graph to be Fano or weak Fano in terms of the graph. Furthermore, we characterize building sets whose associated toric varieties are Fano.

Akiyoshi Tsuchiya

Gorenstein simplices and the associated finite abelian groups

A lattice polytope is a convex polytope each of whose vertices has integer coordinates. It is known that a lattice simplex of dimension d corresponds a finite abelian subgroup of $(\mathbb{R}/\mathbb{Z})^{d+1}$. Conversely, given a finite abelian subgroup of $(\mathbb{R}/\mathbb{Z})^{d+1}$ such that the sum of all entries of each element is an integer, we can obtain a lattice simplex of dimension d. In this talk, we discuss a characterization of Gorenstein simplices in terms of the associated finite abelian groups. Gorenstein polytopes are of interest in combinatorial commutative algebra, mirror symmetry and tropical geometry. In particular, we present complete characterizations of Gorenstein simplices whose normalized volume equal p, p^2 and pq, where p, q are prime integers.

Hailun Zheng

A characterization of simplicial manifolds with $g_2 \leq 2$

The celebrated low bound theorem states that any simplicial manifold of dimension ≥ 3 satisfies $g_2 \geq 0$, and equality holds if and only if it is a stacked sphere. Furthermore, more recently, the class of all simplicial spheres with $g_2 = 1$ was characterized by Nevo and Novinsky, by an argument based on rigidity theory for graphs. In this talk, I will first define three different retriangulations of simplicial complexes that preserve the homeomorphism type. Then I will show that all simplicial manifolds with $g_2 \leq 2$ can be obtained by retriangulating a polytopal sphere with a smaller g_2. This implies Nevo and Novinskys result for simplicial spheres of dimension ≥ 4. More surprisingly, it also implies that all simplicial manifolds with $g_2 = 2$ are polytopal spheres.