ALGEBRAIC AND GEOMETRIC COMBINATORICS ON LATTICE POLYTOPES

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Per Alexandersson

The integer decomposition property and Gelfand-Tsetlin polytopes

We examine the integer decomposition property (IDP) of Gelfand-Tsetlin polytopes. The integral property and the IDP have a nice interplay with a certain partial order on the family of GT polytopes. Furthermore, in the case corresponding to standard Young tableaux, we can completely classify which polytopes are integral, and in this case they also have a regular unimodular triangulation.

If there is time, we also discuss some large counterexamples to natural conjectures.

Elie Alhajjar

Numerical Semigroups and Kunz Polytopes

A numerical semigroup S is an additive submonoid of $\mathbb{N}$ whose complement is finite. The cardinality of $\mathbb{N}_0 \setminus S$ is called the genus of S and is denoted by $g(S)$. The first nonzero element of S is called the multiplicity of S and is denoted by $m(S)$. In this talk, we focus on the number $N(m, g)$ of numerical semigroups with parameters m and g. It is known that $N(m, g)$ can be formulated as the number of integer points in a certain family of rational polytopes and hence coincides with a quasi-polynomial in g of degree $m - 2$. We show that the leading coefficient is constant and provide an interpretation for it. Moreover, we relate $N(m, g)$ to the number $MED(m, g)$ of maximally embedded numerical semigroups with the same parameters. We finish the talk by discussing the special case $m = 4$.

Gabriele Balletti

How connected are the skeletons of polytopes?

Connectivity of a graph measures how easy it is to disconnect a graph by deleting vertices. For example, we say that a cycle is 2-connected because you cannot disconnect it by removing less than 2 vertices. This generalizes to the skeletons of polytopes: a result by Balinsky (1961) shows that you need to remove at least d vertices to disconnect the skeleton of a d-dimensional polytope. Barnette (1982) generalized this result to simplicial pseudomanifolds, and Athanasiadis (2011) proved that you need to remove 2d vertices in order to disconnect the skeleton of a flag d-dimensional simplicial pseudomanifold. Using commutative algebra tools, we show a surprisingly easy proof of a generalization of those results.

Mónica Blanco

Lattice 3-polytopes: quantum jumps and interior points

For P and Q lattice 3-polytopes, the pair (P, Q) is a quantum jump if $P \subset Q$ and $Q \setminus P$ contains exactly one lattice point. In this talk I will put together all we know on quantum jumps in dimension 3, and which information this gives us on lattice 3-polytopes with interior lattice points.

Christopher Borger

The mixed volume and the mixed degree of a family of lattice polytopes

The mixed volume of n-dimensional lattice polytopes $P_1, \ldots, P_n$ can be seen as a generalization of the lattice volume of a single lattice polytope. It has a natural algebraic interpretation as it equals to the generic number of solutions of a system of Laurent polynomials $f_1, \ldots, f_n$ with associated Newton polytopes $P_1, \ldots, P_n$. In a similar way the mixed degree is a quite recent proposal of a generalization of the lattice degree of a single polytope to a family of lattice polytopes. We give an insight into the various questions regarding these two notions such as classifications in fixed dimensions, relations among mixed volumes of certain families and deriving upper bounds for the volume of the Minkowski sum $P_1 + \cdots + P_n$ in terms of the mixed volume of $P_1, \ldots, P_n$.

Federico Castillo

Deformation cones for polytopes

Given a lattice polytope, the set of all polytopes having the same (or a coarsening) normal fan is a polyhedral cone. This cone has appeared in different contexts, for example it is closely related to the nef cone of the associated toric variety. In the case of the regular permutohedron we get the cone of submodular functions. The purpose of this talk is to survey known results and show how to compute this deformation cones in further combinatorial examples. This is joint work with Fu Liu, Federico Ardila, and Alex Postnikov.

Daniel Cavey

Classification of Minimal Polygons with Specified Singularity Content

It is known that there are only finitely many mutation-equivalence classes with a given singularity content, and each of these equivalence classes contains only finitely many minimal polygons. In this talk I describe the notions of mutation and singularity content, and their relevance to the classification of del Pezzo surfaces admitting a toric degeneration.

Giulia Codenotti

The covering minima of lattice polytopes

The covering minima of a convex body were introduced by Kannan and Lovasz to give a better bound in the flatness theorem, which states that lattice point free convex bodies cannot have arbitrarily large width. These minima are similar in flavor to Minkowski's successive minima, and on the other hand generalize the covering radius of a convex body. I will speak about ongoing joint work with Francisco Santos and Matthias Schymura, where we investigate extremal values of these covering minima for lattice polytopes.

Brian Davis

Predicting the Integer Decomposition Property via Machine Learning

In this talk we describe the use of neural networks in approximating algebraic properties associated to lattice simplices. In particular we predict the distribution of Hilbert basis elements in the fundamental parallelepiped, from which we detect the Integer Decomposition Property (IDP). We discuss the results of the approximation method when scanning very large test sets for examples of IDP simplices faster than is possible when computing with Normaliz.

Laura Escobar

Wall-crossing phenomena for Newton-Okounkov bodies

A Newton-Okounkov body is a convex set associated to a projective variety, equipped with a valuation. These bodies generalize the theory of Newton polytopes. Work of Kaveh-Manon gives an explicit link between tropical geometry and Newton-Okounkov bodies. We use this link to describe a wall-crossing phenomenon for Newton-Okounkov bodies. As an application we show how the wall-crossing formula for the tropicalization of Gr(2, n) is an instance of our phenomenon for Newton-Okounkov bodies. Joint work with Megumi Harada.

Magda Hlavacek

Dehn-Sommerville equations for cubical polytopes

The classical Dehn-Sommerville equations, relating the face numbers of simplicial polytopes, have an analogue for cubical polytopes. We show how to obtain these relations by considering the zeta polynomials of the face lattices of cubical polytopes. We also explore how these ideas connect to chain partitions of these face lattices.

Óscar Iglesias Valiño

The complete classification of empty lattice 4-simplices

In previous work we classified all empty 4-simplices of width at least three. We here classify those of width two. There are 2 two-parameter families that project to the second dilation of a unimodular triangle, 29 + 23 one-parameter families of them that project to hollow 3-polytopes, and 2282 individual ones that do not. These families complete the classification of empty 4-simplices. This is joint work with Francisco Santos.

Lukas Katthän

The lecture hall cone as a toric deformation

The lecture hall cone is a particular simplicial cone whose lattice points correspond to so-called Lecture Hall Partitions. The celebrated Lecture Hall Theorem of Bousquet-Mélou and Eriksson states that the generating function of these partitions factors in a very nice way. Over the years, several proofs of this result have been found, but it is still not considered to be well-understood from a geometric perspective. In my talk I am going to present a conjecture where the Ehrhart ring of the Lecture Hall cone is realized as toric deformation of a simpler ring. In particular, this conjecture would imply the Lecture Hall theorem.

Florian Kohl

Levelness of Order Polytopes

Since their introduction by Stanley order polytopes have been intriguing mathematicians as their geometry can be used to examine (algebraic) properties of finite posets. In this talk, we follow this route to examine the levelness property of order polytopes. The levelness property was also introduced by Stanley and it generalizes the Gorenstein property. This property has been recently characterized by Miyazaki for the case of order polytopes. We provide an alternative characterization using weighted digraphs. Using this characterization, we give a new infinite family of level posets and show that determining levelness is in co-NP. This family can be used to create infinitely many examples illustrating that the levelness property can not be characterized by the h^*-vector. This is joint work with Christian Haase and Akiyoshi Tsuchiya.

Yonggyu Lee

Geometrical structure of Tesler polytopes

Tesler polytope is the set of upper triangular matrices with non negative entries satisfying certain equations called hook sum conditions. We will show that the Tesler polytopes of positive hook sums are all combinatorially isomorphic to each other and discuss about how to apply this result to the conjecture of Ehrhart positivity of Tesler polytope of hook sum $(1, \dots, 1)$. This is joint work with Fu Liu.

Sebastian Manecke

Gram's relation for cone valuations

We study linear relations of interior and exterior angle vectors of polytopes, where we generalize the usual solid angle with simple cone valuations. For both interior and exterior angles, we prove that only one linear relation exists for any cone angle, one being Gram's relation.

The uniqueness of the relation follows from a connection between angle sums and the combinatorics of zonotopes. The angle sum vectors are given by the Co-Whitney numbers of the lattice of flats associated to the zonotope. This shows that surprisingly the angle-sum vectors of zonotopes are independent of the notion of angle used.

This is joint work with Spencer Backman and Raman Sanyal.

Andrés Vindas Meléndez

Fixed Subpolytopes of the Permutahedron

Motivated by the generalization of Ehrhart theory with group actions, this project makes progress towards obtaining the equivariant Ehrhart theory of the permutahedron. The fixed subpolytopes of the permutahedron are the polytopes that are fixed by acting on the permutahedron by a permutation. We prove some general results about the fixed subpolytopes. In particular, we compute their dimension, show that they are combinatorially equivalent to permutahedra, provide hyperplane and vertex descriptions, and prove that they are zonotopes. Lastly, we obtain a formula for the volume of these fixed subpolytopes, which is a generalization of Richard Stanley's result of the volume for the standard permutahedron. This is joint work with Federico Ardila (San Francisco State) and Anna Schindler (University of Washington).

Makoto Miura

Hibi toric varieties and mirror symmetry

A Hibi toric variety is the projective toric variety corresponding to the order polytope of a finite poset, which equips rich combinatorial descriptions inherited from the poset. The varieties also appear as the special fibers of certain toric degenerations of Grassmannians or some other Gorenstein Fano varieties. This fact is helpful to study the geometry of complete intersection Calabi–Yau 3-folds in such Fano varieities and to construct conjectural mirror manifolds for them via conifold transitions.

Our aim in this talk is to see how well the combinatorics of Hibi toric varieties works for the study of certain kinds of Calabi–Yau 3-folds and their mirror symmetry.

Louis Ng

Magic Counting with Inside-Out Polytope

In this paper, we investigate strong 4×4 pandiagonal magic squares (no entries repeat; equal row sums, column sums, and diagonal sums, including the wraparound ones), strong 5×5 pandiagonal magic squares, weak $2 \times n$ magic rectangles (repeating entries allowed; equal row sums and equal column sums), and $2 \times n$ magilatin rectangles (no entries repeat in a row/column; equal row sums and equal column sums). The magic counts depend on the magic sum or the upper bound of the entries. We compute their counting quasipolynomials and the associated generating functions by using a geometrical interpretation of the problems, considering them as counting the lattice points in polytopes with removed hyperplanes.

Jorge Alberto Olarte

Transversal valuated matroids and their presentation space

The tropical Stiefel map sends a matrix with tropical entries to the valuated matroid given by its maximal tropical minors. We say that a valuated matroid is transversal if it arises this way. We show that this is indeed a generalization of transversal matroids and that a valuated matroid is transversal if and only if the all the facets of the polytopal complex dual to the corresponding tropical linear space are polytopes of transversal matroids. Given a transversal valuated matroid, we call a matrix in the preimage under the tropical Stiefel map a presentation and the preimage itself the space of presentations. The rows of any presentation are always points in the corresponding tropical linear space. So we show how to choose points in the tropical space such that they form the rows of a presentation and that the space of presentations is a product of fans modulo permutation of rows.

McCabe Olsen

Level algebras and lecture hall polytopes

Given a family of lattice polytopes, a common question in Ehrhart theory is classifying which polytopes in the family are Gorenstein. A less common question is classifying which polytopes in the family admit level semigroup algebras, a generalization of Gorenstein algebras. In this article, we consider these questions for $\boldsymbol{s}$-lecture hall polytopes, which are a family of simplices arising from lecture hall partitions. We provide a complete characterization of the Gorenstein property for lecture hall polytopes. We provide a different, more geometric characterization of the Gorenstein property which applies to a large subfamily of lecture hall polytopes. Additionally, we also give a complete characterization for the level property in terms of $\boldsymbol{s}$-inversion sequences and demonstrate some consequences of the classification result. This is joint work with Florian Kohl.

Marta Panizzut

K3 polytopes and their quartic surfaces

The closure of the connected components of the complement of a tropical hypersurface are called regions. The have the structure of convex polyhedra and they might be bounded or unbounded. A 3-dimensional polytope is a K3 polytope if it is the closure of the bounded region of a smooth tropical quartic surface.

In this talk we begin by studying properties of K3 polytopes. In particular we exploit their duality to regular unimodular central triangulations of reflexive polytopes in the fourth dilation of the standard tetrahedron. Then we focus on quartic surfaces that tropicalize to K3 polytopes, and we look at them through the lenses of Geometric Invariant Theory.

This is a joint work with Gabriele Balletti and Bernd Sturmfels.

Irem Portakal

Torus actions on matrix Schubert varieties

Matrix Schubert Varieties were first studied by Fulton in 1990s. He gave a nice combinatorial description of these varieties in terms of Rothe diagrams. Our aim is to understand the natural effective torus action on these varieties. It turns out that this can be achieved by bipartite graphs. In particular, for the toric case, we describe the face structure of the polyhedral cone in terms of graphs.

Maren Ring
McMullen's formulas for Ehrhart coefficients

The Ehrhart polynomial counts the number of lattice points in dilates of a polytope and gives us a useful connection between volumes and discrete points. Determining the coefficients of this polynomial can be done via so-called McMullen's formulas (aka local formulas), which give the i-th coefficient as a weighted sum of the volumes of the i-dimensional faces of the polytope. Based on choices of lattice cells we construct a new local formula that provides us with additional knowledge about Ehrhart coefficients and can be used for the exploitation of polyhedral symmetries. This is joint work with A. Schürmann.

Benjamin Schröter
Reconstruction of Matroid Polytopes

Blind and Mani, and later Kalai, showed that the face lattice of a simple polytope is reconstructable by its 1-skeleton, i.e., determined by its vertex-edge graph. Joswig extended this result to special cubical polytopes and recently Doolittle et al. presented explicit sufficient conditions when a polytope is reconstructable from its k-skeleton in terms of non-simple vertices. In contrast to all these results, a construction of Perles shows that for every $d > 3$ there are d-polytopes which are not reconstructable from their $(d-3)$-skeleta.

In my talk I will focus on matroid polytopes which are a particular class of 0/1-polytopes. Matroids are fundamental combinatorial objects as they encode an abstract version of dependency and independency. Maximal independent sets satisfy a basis-exchange. An edge in the basis-exchange graph corresponds to such an exchange and the basis-exchange graph is exactly the vertex-edge graph of the matroid polytope. I show that the basis-exchange graph almost determines a matroid and hence a matroid polytope is reconstructable from its graph.

This is work in progress with Guillermo Pineda-Villavicencio.

Liam Solus

Real-rootedness, Unimodality, and Symmetric Decompositions of Polynomials

A central line of research in algebraic and geometric combinatorics concerns itself with the investigation of when certain polynomials with nonnegative integer coefficients are unimodal; i.e. when their sequence of coefficients is unimodal. One way to prove such a polynomial is unimodal is to show that it has only real roots. A second way is to decompose it as a sum of unimodal polynomials whose unimodality in turn implies that of the original polynomial. In this talk, we will combine these two approaches and discuss how real-rootedness techniques play nicely with some decompositions of polynomials into symmetric polynomials. We will then apply these techniques to extend some results and answer some open questions on various families of Ehrhart h^*-polynomials, local h-polynomials, and h-polynomials of simplicial complexes.

Johanna Steinmeyer

No small Counterexamples to Perles' Conjecture

In a celebrated theorem, Blind and Mani proved that the entire combinatorial structure of a simple polytope is determined by its vertex-edge graph. Shortly after, Kalai gave an elegant and constructive proof of the same result. Looking to generalize this, Perles asked the following question: In the graph of a simple d-polytope, is every (d-1)-regular, connected, induced, non-separating subgraph the graph of a facet? This conjecture was disproved by Haase and Ziegler, but it remains unknown how to locally characterize those subgraphs that correspond to the facets of the polytope. We discuss a systematic approach to showing that certain classes of graphs can not yield a counterexample to Perles' Conjecture when they occur as subgraphs.

Yusuke Suyama

Toric Fano varieties associated to graph cubeahedra

The graph cubeahedron of a finite simple graph is a smooth polytope obtained from a cube by truncating the faces corresponding to connected induced subgraphs in increasing order of dimension. In this talk, we give a necessary and sufficient condition for the nonsingular projective toric variety associated to the graph cubeahedron to be Fano or weak Fano in terms of the graph.

Bach Tran

On k-normality and Regularity of Normal Projective Toric Varieties

We will give a bound for a very ample lattice polytope to be k-normal. Equivalently, we give a new combinatorial bound for the Castelnuovo-Mumford regularity of normal projective toric varieties

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Akiyoshi Tsuchiya

Cayley sums and Minkowski sums of 2*-convex-normal lattice polytopes*

In this talk, we discuss the integer decomposition property for Cayley sums and Minkowski sums of lattice polytopes. In particular, we focus on these constructions arising from 2-convex-normal lattice polytopes. Moreover, we discuss the level property of Cayley sums and Minkowski sums.

Lena Walter

The Kingman Coalescent as a Density on a Space of Trees

Randomly pick n individuals from a population and trace their genealogy backwards in time until you reach the most recent common ancestor. The Kingman n-coalescent is a probabilistic model for the tree one obtains this way. The common definitions are stated from a stochastic point of view. The Kingman Coalescent can also be described by a probability density function on a space of certain trees. For the space of phylogenetic trees Ardila and Klivans showed, that this space is closely related to the Bergman fan of the graphical matroid of the complete graph. Using its fine fan structure, similar things can be said about the space we are considering. I will describe the density and report on work in progress with Christian Haase about relations of population genetics and algebraic geometry via the tropical connection.

Hailun Zheng

A lower bound theorem for centrally symmetric simplicial polytopes

An important invariant in the study of face numbers of simplicial d-polytopes is the g-vector. The generalized lower bound theorem states that $g_i \geq 0$ for any simplicial polytope and characterizes the case of equality. Much less is known for centrally symmetric polytopes. A seminal work is established by Stanley thirty years ago, where he proved that for any centrally symmetric simplicial d-polytope P with $d \geq 3$ and $1 \leq i \leq d/2$, $g_i(P) \geq \binom{d}{i} - \binom{d}{i-1}$. In this talk, I will introduce the rigidity theory of frameworks, and show how to apply this machinery to give a characterization of centrally symmetric d-polytopes with $d \geq 4$ that satisfy $g_2 = \binom{d}{2} - d$. This is joint work with Steve Klee, Eran Nevo and Isabella Novik.